"BURNOUT IMPULSE" MAY BE THRUST MISALIGNMENT IN DISGUISE

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Nomenclature

$\vec{M}$ = thrust misalignment torque

$\frac{d}{dt}$ = total derivative

$\frac{d^*}{dt}$ = derivative with respect to rotating coordinate system

$\vec{\omega}$ = angular velocity vector

$\vec{H}$ = angular momentum vector

I_P = pitch moment of inertia

I_R = roll (or spin) moment of inertia

1-2-3 coordinates - body fixed system with the 3-axis being the spin axis.

Coning (or torque-free precession) of spin stabilized spacecraft, commencing abruptly at last stage burnout has frequently been observed and often attributed to a mysterious "burnout impulse." It will be shown here how thrust misalignment, present throughout the burn, may be the real culprit.

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Thomson[1] presents equations of motion for a spinning rocket which include the effects of jet damping and inertia changes. For a symmetric rocket and a purely transverse thrust misalignment torque whose direction defines the body 1-axis, these are of the form:

$$I_P \dot{\omega}_1 = M + (I_P - I_R)\,\omega_3 \omega_2 - \alpha\,\omega_1 \qquad (1)$$

$$I_P \dot{\omega}_2 = \quad - (I_P - I_R)\,\omega_3 \omega_1 - \alpha\,\omega_2 \qquad (2)$$

$$I_R \dot{\omega}_3 = \qquad - \beta\,\omega_3 \qquad (3)$$

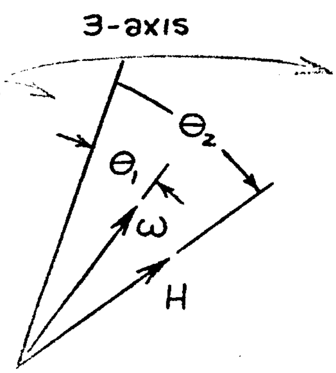
When applying these equations to practical cases of spin stabilized last stage firings α is positive[2], and quasi-steady state behavior is found. ω_3 increases or decreases slightly during the burn, depending on the sign of β, but is nearly a constant of the motion, and after initial transient oscillations, ω_1 and ω_2 settle down to nearly constant values. Thus the system angular velocity vector in the rotating frame becomes essentially constant, and it follows that it is also fixed inertially since, using Symon's[3] notation,

BOLDFACE SMALL OMEGAS

$$\frac{d}{dt}\vec{\omega} = \frac{d^*}{dt}\vec{\omega} + \vec{\omega} \times \vec{\omega} = 0.$$

BOLDFACE OMEGA AND ITAL H

Now, for a symmetric body, the 3-axis, $\vec{\omega}$, and $\vec{H}$ are coplanar and we have



$$\tan\theta_1 = \frac{(\omega_1^2 + \omega_2^2)^{1/2}}{\omega_3} \qquad (4)$$

$$\tan\theta_2 = \frac{I_P(\omega_1^2 + \omega_2^2)^{1/2}}{I_R\,\omega_3}$$

$$= \frac{I_P}{I_R}\tan\theta_1 \qquad (5)$$

where ω_1 and ω_2 are calculated from equations (1) and (2) with $\dot{\omega}_1 = \dot{\omega}_2 = 0$.

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Looking now down the $\vec{\omega}$ vector, one would see the tips of the spin axis and the momentum vector moving in circles whose radii might be represented by θ_1 and $(\theta_2 - \theta_1)$ respectively as shown in Figure 1.

Now if $I_P \gg I_R$, which is often the case, θ_1 is much smaller than θ_2 so the spin axis motion during the burn is quite limited. At burnout[4] however, the momentum vector stops in space and torque-free precession of the spin about it begins (see Figure 2).

This is the coning which could be erroneously attributed to a "burnout impulse." It is interesting to note that thrust misalignment can change the final spacecraft attitude significantly while leaving rocket performance very nearly nominal.

[1] Thomson, W. T., Introduction to Space Dynamics (John Wiley & Sons, Inc., New York, 1961), page 225.

[2] In many practical cases, $\alpha \approx \dot{m}\ell^2$ where $\dot{m}$ is the mass flow rate, and ℓ is the distance from the center of mass to the end of the rocket nozzle.

[3] Symon, K. R., Mechanics (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1961), 2nd ed., page 276.

[4] The construction presented assumes instantaneous burnout. A finite thrust decay results in a decrease of the cone angle.

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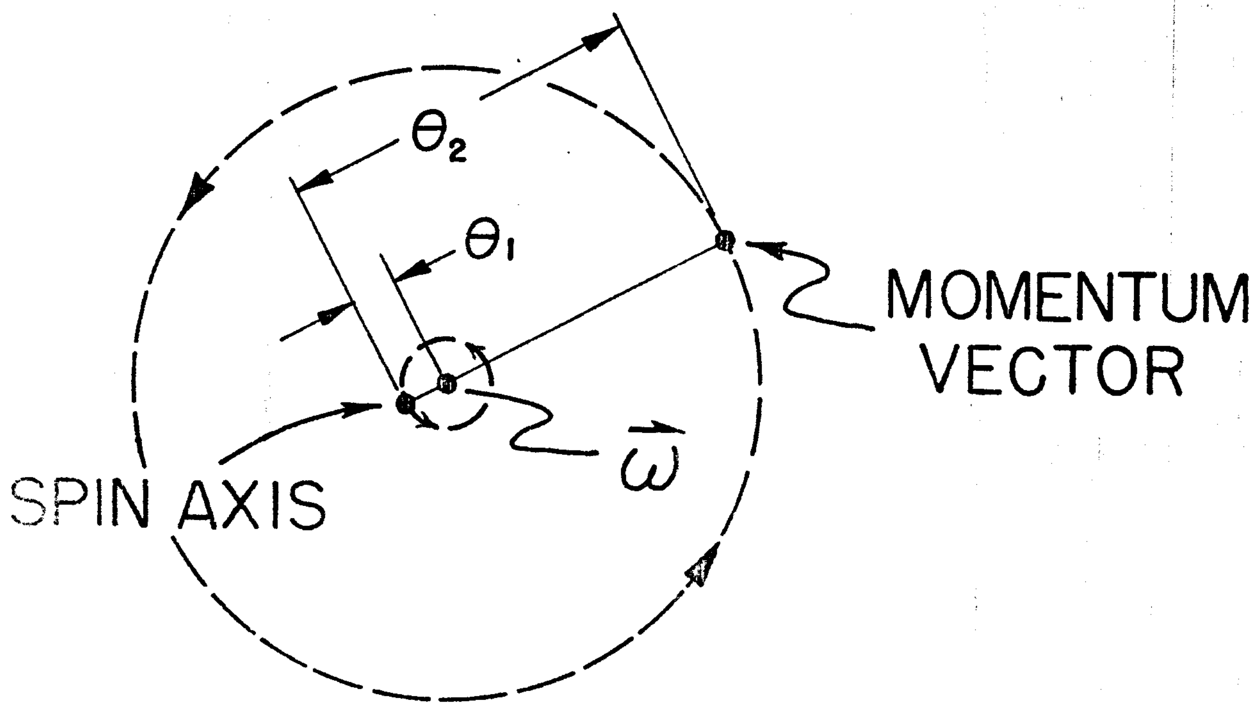

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FIG. 1

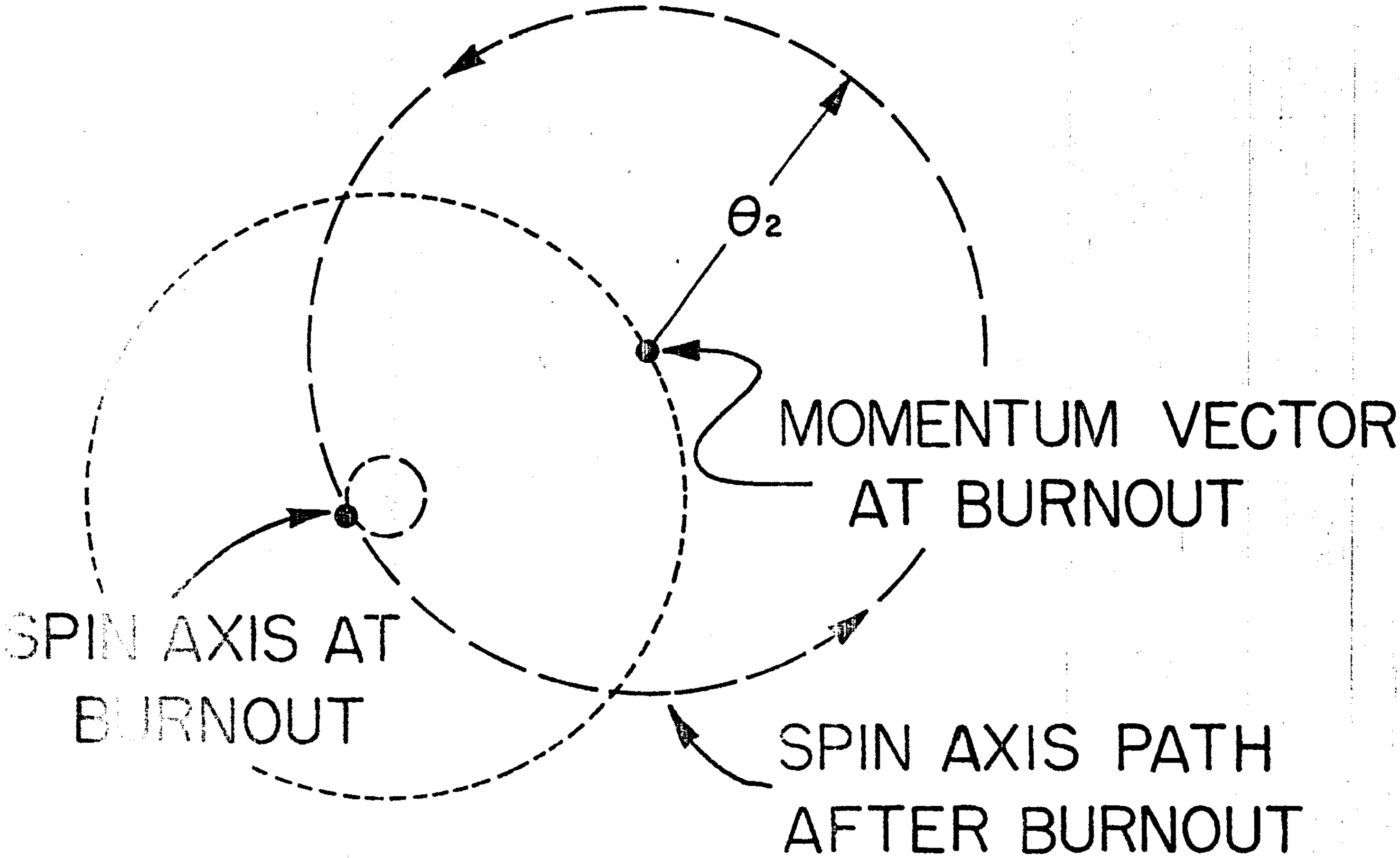

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FIG. 2